Published by Institute of Physics Publishing for SISSA/ISAS
Received: *March 5, 2003*
Accepted: *April 4, 2003*

Modified mode-expansion on a BPS wall related to the nonlinear realization

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Abstract: We propose a modified mode-expansion of the bulk fields in a BPS domain wall background to obtain the effective theory on the wall. The broken SUSY is nonlinearly realized on each mode defined by our mode-expansion. Our work clarifies a relation between two different approaches to derive the effective theory on a BPS wall, i.e. the nonlinear realization approach and the mode-expansion approach. We also discuss a further modification that respects the Lorentz and $\mathrm{U}(1)_R$ symmetries broken by the wall.

Keywords: Solitons Monopoles and Instantons, Supersymmetric Effective Theories, Superspaces.

Contents

1. Introduction

In supersymmetric (SUSY) theories, there are important states called BPS states [1]. They preserve part of the supersymmetry of the theory and play a crucial role in quantum field theories. One of the simplest example of BPS states is a BPS domain wall. In particular, BPS domain walls in four-dimensional ($4D$) $\mathcal{N} = 1$ SUSY theories have been thoroughly investigated in a number of papers because such theories are tractable and have various types of BPS walls with interesting features [2]–[6].

Besides that BPS domain walls are an intriguing subject to research in their own rights, they are also important in the brane-world scenario [7]–[9] because they can provide a natural realization of $5D$ $\mathcal{N} = 1$ SUSY (eight supercharges) to $4D$ $\mathcal{N} = 1$ SUSY (four supercharges), which is relevant to the phenomenology. Of course, since our world is four-dimensional, we should discuss a domain wall in $5D$ theories for the realistic model-building.

However, $5D$ SUSY theory is quite restrictive and difficult to handle. So it is useful and instructive to study BPS walls in $4D$ $\mathcal{N}=1$ theories as a toy model. In this paper, we will concentrate ourselves on BPS walls in the $4D$ $\mathcal{N}=1$ generalized Wess-Zumino model for simplicity. To discuss the physics in the BPS-wall background, it is useful to investigate a low-energy effective theory on the wall. Since a BPS wall preserves a half of the bulk SUSY, such an effective theory can be expressed by $3D$ superfields. There are mainly two approaches to derive the effective theory described by $3D$ superfields.

The first one is the nonlinear realization approach [10, 11]. From the $3D$ viewpoint, SUSY breaking by a BPS wall can be regarded as the partial SUSY breaking from $3D$ $\mathcal{N}=2$ to $\mathcal{N}=1$. Thus, we can obtain the effective theory on the wall by constructing an invariant action under the broken SUSY, which is realized nonlinearly. This approach is useful for the discussion of the general properties of BPS supermembranes since this method uses only information on the breaking pattern of the symmetries. This also means, however, that we cannot determine parameters of the effective theory in this approach.

The second one is the mode-expansion approach [12]. In this approach, the $3D$ effective theory is directly derived from the original $4D$ theory. Specifically, we expand the fluctuation fields around the wall-background into an infinite number of $3D$ superfields, and integrating out the heavy modes. In this approach, we can see explicitly how the $3D$ superfields in the effective theory are embedded into the original $4D$ superfields. Unlike the previous approach, parameters of the effective theory are obtained as the overlap integrals of the background field configuration and the mode functions. However, this approach does not respect the symmetries broken by the wall.

These two approaches are complementary to each other. So it is useful and instructive to clarify the relation between them. This is the purpose of this paper. Specifically, we will propose the modification of the naive mode-expansion of the bulk superfields so that the broken SUSY is nonlinearly realized on each mode. Using our modified mode-expansion, we can obtain an invariant effective action under the broken SUSY, and can also calculate parameters of the effective theory. In the latter part of this paper, we try to modify the mode-expansion further so that it also respects the Lorentz and $\mathrm{U}(1)_R$ symmetries broken by the wall.

The paper is organized as follows. In the next section, we will provide a brief review of the nonlinear realization approach. In section 3, we will review our previous work [12] where a naive mode-expansion is discussed. Then, in section 4, we will modify it so that the broken SUSY is nonlinearly realized on each mode. In section 5, we will discuss the further modification of the mode-expansion which also respects the broken Lorentz and $\mathrm{U}(1)_R$ symmetries. Section 6 is devoted to the summary and the discussion. Notations and some formulae are listed in the appendices.

2. Review of the nonlinear realization approach

In this section, we will briefly review the nonlinear realization approach to construct an effective action for the supermembrane [10]. Throughout this paper, we will assume that the background space-time is flat and has a $4D$ $\mathcal{N}=1$ supersymmetry.

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From the $3D$ viewpoint, the $4D$ $\mathcal{N}=1$ SUSY algebra is a central extended $\mathcal{N}=2$ SUSY algebra.

$$\begin{aligned}\{Q_{1\alpha},Q_{1\beta}\} &= \{Q_{2\alpha},Q_{2\beta}\}=2(\gamma^m_{(3)}\sigma^2)_{\alpha\beta}P_m\,,\\ \{Q_{1\alpha},Q_{2\beta}\} &= -\{Q_{2\alpha},Q_{1\beta}\}=2i(\sigma^2)_{\alpha\beta}P_2\,,\end{aligned} \tag{2.1}$$

where $m=0,1,3$ denotes the $3D$ space-time index, and α,β denote the $3D$ Majorana spinor indices.[1]

The existence of the BPS membrane breaks the bulk symmetry $G=\{P_m,P_2,Q_{1\alpha},Q_{2\alpha}\}$ to the vacuum stability subgroup $H=\{P_m,Q_{2\alpha}\}$. Then, a coset element Ω can be parameterized as

$$\Omega = e^{ix^mP_m+\theta_2Q_2}e^{i\kappa\rho_0P_2+\kappa\zeta_0Q_1}\,. \tag{2.2}$$

Here $\rho_0=\rho_0(x^m,\theta_2)$ and $\zeta_{0\alpha}=\zeta_{0\alpha}(x^m,\theta_2)$ are scalar and spinor superfields corresponding to the Nambu-Goldstone (NG) modes for P_2 and $Q_{1\alpha}$ respectively, and κ is a constant[2] whose mass-dimension is $-3/2$.

The transformation laws of each superfield for the broken symmetries can be read off as follows by multiplying Ω by corresponding group elements from the left.

$$\begin{aligned}\delta^{P_2}_a\rho_0 &= a\,,\\ \delta^{P_2}_a\zeta_{0\alpha} &= \delta^{P_2}_a\phi=0\,,\end{aligned} \tag{2.3}$$

$$\begin{aligned}\delta^{Q_1}_{\xi_1}\rho_0 &= -2\kappa^{-1}\xi_1\theta_2-i\kappa\xi_1\gamma^m_{(3)}\zeta_0\partial_m\rho_0\,,\\ \delta^{Q_1}_{\xi_1}\zeta_{0\alpha} &= \kappa^{-1}\xi_{1\alpha}-i\kappa\xi_1\gamma^m_{(3)}\zeta_0\partial_m\zeta_{0\alpha}\,,\\ \delta^{Q_1}_{\xi_1}\phi &= -i\kappa\xi_1\gamma^m_{(3)}\zeta_0\partial_m\phi\,,\end{aligned} \tag{2.4}$$

where a and $\xi_{1\alpha}$ are transformation parameters, and ϕ denotes a matter superfield. Indeed, the above transformations satisfy the SUSY algebra (2.1). In this paper, we will call the above transformation laws for the broken symmetries the *standard non-linear transformations.*

Note that the NG superfields ρ_0 and ζ_0 introduced by eq. (2.2) are not independent of each other, because the NG modes for P_2 and Q_1 form a supermultiplet for the unbroken Q_2-SUSY. The relation between them can be obtained by setting a covariant constraint (inverse Higgs effect [13]),

$$\mathcal{D}_{2\alpha}\rho_0=0\,, \tag{2.5}$$

where $\mathcal{D}_{2\alpha}$ is a covariant spinor derivative in the presence of the NG superfields, and its explicit form is listed in eq. (A.9) in appendix A.1. From this constraint, we can express $\zeta_{0\alpha}$ in terms of ρ_0.

$$\zeta_{0\alpha}=-\frac{1}{2}D_{2\alpha}\rho_0+\mathcal{O}(\kappa^2)\,. \tag{2.6}$$

Namely, the essential NG superfield is $\rho_0(x^m,\theta_2)$ only.

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[1] In this paper, we will choose the x_2-direction to be perpendicular to the membrane or the domain wall.

[2] The mass scale $f\equiv\kappa^{-2/3}$ corresponds to the scale where P_2 and Q_1 are broken, and $f^3=\kappa^{-2}$ is a tension of the membrane.

By using ρ_0, we can construct a $3D$ $\mathcal{N}=2$ invariant action perturbatively for κ by the standard procedure of the nonlinear realization [11].

For an invariant action for the NG modes $S_{\rm NG}$, there is an alternative method to derive the effective action for all orders in κ [10, 14]. We will explain this method in the following.

Let us introduce a scalar NG superfield $\varphi_0(x^m,\theta_2)$. As will be shown, φ_0 coincides with ρ_0 at the lowest order in κ. Then, define a spinor superfield $\psi_{0\alpha}(x^m,\theta_2)$ as

$$\psi_{0\alpha} \equiv -\frac{1}{2}D_{2\alpha}\varphi_0\,. \tag{2.7}$$

This means that ψ_0 satisfies the following constraint.

$$D_2^2\psi_{0\alpha} = -2i(\gamma^m_{(3)}\partial_m\psi_0)_\alpha\,. \tag{2.8}$$

Noticing that $\psi_{0\alpha}$ is the NG superfield for Q_1, we can write its Q_1-transformation law which preserves the constraint (2.8) and satisfies the SUSY algebra (2.1) as

$$\delta^{Q_1}_{\xi_1}\psi_{0\alpha} = \kappa^{-1}\xi_{1\alpha} + \frac{\kappa}{4}\xi_{1\alpha}D_2^2\Xi - \frac{i\kappa}{2}(\gamma^m_{(3)}\xi_1)_\alpha\partial_m\Xi\,. \tag{2.9}$$

Here Ξ is a scalar superfield and transforms under the Q_1 transformation as

$$\delta^{Q_1}_{\xi_1}\Xi = 2\kappa^{-1}\xi_1\psi_0 = -\kappa^{-1}\xi_1 D_2\varphi_0\,. \tag{2.10}$$

From eqs. (2.9) and (2.10), we can see that Ξ satisfies the following recursive equation.

$$\Xi = \frac{\psi_0^2}{1+\frac{\kappa^2}{4}D_2^2\Xi}\,. \tag{2.11}$$

This equation can be solved and the solution is

$$\Xi = \frac{2\psi_0^2}{1+\sqrt{1+\kappa^2 D_2^2(\psi_0^2)}}\,. \tag{2.12}$$

From the transformation law (2.10), we can see that

$$S_{\rm NG} = \int \mathrm{d}^3x\mathrm{d}^2\theta_2\; 2\Xi \tag{2.13}$$

is Q_1-invariant. So this is a good candidate for the effective action of the NG modes. The right-hand-side of eq. (2.13) is expanded by κ as

$$S_{\rm NG} = \int \mathrm{d}^3x\mathrm{d}^2\theta_2\,\left\{\frac{1}{2}(D_2\varphi_0)^2 + \mathcal{O}(\kappa^2)\right\}. \tag{2.14}$$

This certainly is an action for a massless superfield. Actually, after eliminating the auxiliary field of φ_0, the bosonic part of $S_{\rm NG}$ becomes the Nambu-Goto action in the static gauge. (See appendix B.)

Now we define the quantity

$$E \equiv \frac{1}{1+\frac{\kappa^2}{4}D_2^2\Xi}\,. \tag{2.15}$$

Then the relation between $\psi_{0\alpha}$ and $\zeta_{0\alpha}$ can be written by

$$\zeta_{0\alpha} = \frac{\psi_{0\alpha}}{E} \,. \tag{2.16}$$

In fact, we can easily show that the right-hand-side of this equation transforms as $\zeta_{0\alpha}$ in eq. (2.4) under the Q_1-SUSY.

From eq. (2.16), we can see that

$$\psi_{0\alpha} = \zeta_{0\alpha} + \mathcal{O}(\kappa^2) \,. \tag{2.17}$$

This means that

$$\varphi_0 = \rho_0 + \mathcal{O}(\kappa^2) \,, \tag{2.18}$$

as mentioned before.

Using E defined by eq. (2.15), an invariant action for a matter field ϕ can be written in the form of

$$S_{\rm matter} = \int \mathrm{d}^3x \mathrm{d}^2\theta_2 \, E\mathcal{F}(\phi, \mathcal{D}_{2\alpha}\phi, \mathcal{D}_m\phi, \ldots) \,, \tag{2.19}$$

where $\mathcal{F}$ is a $3D$ Lorentz invariant function of ϕ and its covariant derivatives.

From eqs. (2.7) and (2.9), we can read off the Q_1-transformation law of φ_0 as

$$\delta^{Q_1}_{\xi_1}\varphi_0 = -2\kappa^{-1}\xi_1\theta_2 + \kappa\xi_1 D_2 \Xi \,. \tag{2.20}$$

If we define a complex scalar superfield

$$\phi_0 \equiv \kappa\varphi_0 + i\kappa^2\Xi \,. \tag{2.21}$$

Eqs. (2.10) and (2.20) are collectively written as[3]

$$\delta^{Q_1}_{\xi_1}\phi_0 = -2\xi_1\theta_2 - i\xi_1 D_2\phi_0 \,. \tag{2.22}$$

The P_2-transformation law of ϕ_0 can be defined as

$$\delta^{P_2}_a \phi_0 = a \,. \tag{2.23}$$

Eqs. (2.22) and (2.23) form the SUSY algebra (2.1).

3. Mode-expansion approach

Now we will discuss the mode-expansion approach to obtain the effective action on the BPS wall. Here we will consider the $4D$ $\mathcal{N}=1$ generalized Wess-Zumino model as a bulk theory. The action is

$$S = \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \, K(\Phi, \bar{\Phi}) + \int \mathrm{d}^4x \mathrm{d}^2\theta \, W(\Phi) + \int \mathrm{d}^4x \mathrm{d}^2\bar{\theta} \, \bar{W}(\bar{\Phi}) \,, \tag{3.1}$$

[3] By using ϕ_0, we can construct a linear realization for partial SUSY breaking: $3D$ $\mathcal{N}=2 \to \mathcal{N}=1$ [15].

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where Φ^i ($\bar{\Phi}^{\bar{i}}$) are (anti-) chiral superfields and

$$\Phi^i(y,\theta) = A^i(y) + \sqrt{2}\theta\Psi^i(y) + \theta^2 F^i(y) \qquad (y^\mu \equiv x^\mu + i\theta\sigma^\mu\bar{\theta})\,. \tag{3.2}$$

We assume that this theory has a BPS domain wall $A^i = A^i_{\rm cl}(x_2)$. The classical solution $A^i_{\rm cl}(x_2)$ satisfies the following BPS equation.

$$\partial_2 A^i = e^{i\delta} K^{i\bar{j}} \bar{W}_{\bar{j}}\,, \tag{3.3}$$

where $\partial_2 \equiv \partial/\partial x_2$, and $K^{i\bar{j}}$ is an inverse matrix of the Kähler metric $K_{i\bar{j}} \equiv \partial^2 K/\partial\Phi^i\partial\bar{\Phi}^{\bar{j}}$. Lower indices denote derivatives in terms of corresponding superfields. δ is a phase determined by $A^i_{\rm cl}$,

$$\delta \equiv \arg\left(\int_\Gamma {\rm d}W\right), \tag{3.4}$$

where Γ is an orbit for $A^i_{\rm cl}(x_2)$ on the target space of the scalar field.

Now we decompose the grassmannian coordinates θ and $\bar{\theta}$ as

$$\theta^\alpha = \frac{e^{i\delta/2}}{\sqrt{2}}(\theta_1^\alpha + i\theta_2^\alpha)\,, \qquad \bar{\theta}^{\dot{\alpha}} = \frac{e^{-i\delta/2}}{\sqrt{2}}(\theta_1^\alpha - i\theta_2^\alpha)\,, \tag{3.5}$$

where θ_i^α $(i=1,2)$ are $3D$ Majorana spinors, and corresponding to this, we decompose the supercharges as

$$Q_\alpha = \frac{e^{-i\delta/2}}{\sqrt{2}}(Q_{1\alpha} - iQ_{2\alpha})\,, \qquad \bar{Q}_{\dot{\alpha}} = -(Q_\alpha)^* = -\frac{e^{i\delta/2}}{\sqrt{2}}(Q_{1\alpha} + iQ_{2\alpha})\,. \tag{3.6}$$

Then it follows that

$$\theta Q + \bar{\theta}\bar{Q} = \theta_1 Q_1 + \theta_2 Q_2\,. \tag{3.7}$$

In this case, Q_1 and Q_2 correspond to the broken and the unbroken supercharges, respectively. Under the decomposition (3.6), the $4D$ $\mathcal{N}=1$ SUSY algebra is rewritten as eq. (2.1).

After performing the integration in terms of θ_1, the original action (3.1) becomes [12]

$$S = \int {\rm d}^3x{\rm d}^2\theta_2{\rm d}x_2 \left\{K_{i\bar{j}} D_2^\alpha\varphi^i D_{2\alpha}\bar{\varphi}^{\bar{j}} - 2iK_i\partial_2\varphi^i + 4{\rm Im}\left(e^{-i\delta}W(\varphi)\right)\right\}\,. \tag{3.8}$$

Here

$$\varphi^i(x^m, x_2, \theta_2) \equiv e^{ix^m P_m + ix_2P_2 + \theta_2 Q_2} \times A^i(0)\,, \tag{3.9}$$

where the definition of the action of the generators on the fields $\times$ is given in appendix A.2, and its relation to the $4D$ chiral superfield Φ^i is

$$\Phi^i(x^m, x_2 + \theta_1\theta_2, \theta_1, \theta_2) = e^{-i\theta_1 D_2 + i\theta_1^2\partial_2}\varphi^i(x^m, x_2, \theta_2)\,. \tag{3.10}$$

Using φ^i, we can rewrite the superfield equations of motion

$$-\frac{1}{4}D^2K_i + W_i = 0\,, \tag{3.11}$$

as

$$-\frac{i}{2}\left\{K_{i\bar{j}}D_2^2\bar{\varphi}^{\bar{j}}+K_{i\bar{j}\bar{k}}D_2^\alpha\bar{\varphi}^{\bar{j}}D_{2\alpha}\bar{\varphi}^{\bar{k}}\right\}-K_{i\bar{j}}\partial_2\varphi^{\bar{j}}+e^{-i\delta}W_i=0\,. \tag{3.12}$$

Then, the equations of motion for the fluctuation field $\tilde{\varphi}^i$ around the background $\varphi^i_{\rm cl}=A^i_{\rm cl}(x_2)$ can be obtained by substituting $\varphi^i=\varphi^i_{\rm cl}+\tilde{\varphi}^i$ into eq. (3.12). Using the BPS equation (3.3), it can be written as

$$-\frac{1}{2}K_{i\bar{j}}(A_{\rm cl})D_2^2\bar{\tilde{\varphi}}^{\bar{j}}+i\left\{\mathcal{D}_y\bar{\tilde{\varphi}}_i-e^{-i\delta}\mathcal{D}_iW_j(A_{\rm cl})\tilde{\varphi}^j\right\}+\cdots=0\,, \tag{3.13}$$

where the ellipsis denotes higher terms for $\tilde{\varphi}$, and

$$\bar{\tilde{\varphi}}_i\equiv K_{i\bar{j}}(A_{\rm cl})\bar{\tilde{\varphi}}^{\bar{j}}\,,\qquad \mathcal{D}_y\bar{\tilde{\varphi}}_i\equiv\partial_2\bar{\tilde{\varphi}}_i-\Gamma^k_{ij}(A_{\rm cl})\partial_2A^j_{\rm cl}\bar{\tilde{\varphi}}_k\,,\qquad \mathcal{D}_iW_j\equiv W_{ij}-\Gamma^k_{ij}W_k\,. \tag{3.14}$$

Here $\Gamma^k_{ij}\equiv K^{k\bar{l}}K_{ij\bar{l}}$ is a connection on the Kähler manifold.

From eq. (3.13), we can find the mode equation,

$$i\left\{\mathcal{D}_y\bar{u}_{(n)i}-e^{-i\delta}\mathcal{D}_iW_j(A_{\rm cl})u^j_{(n)}\right\}=m_{(n)}\bar{u}_{(n)i}\,, \tag{3.15}$$

where $\bar{u}_{(n)i}\equiv K_{i\bar{j}}(A_{\rm cl})\bar{u}^{\bar{j}}_{(n)}$.

Using the mode function $u^i_{(n)}(x_2)$, we can expand the $4D$ field φ^i as

$$\varphi^i(x^m,x_2,\theta_2)=A^i_{\rm cl}(x_2)+\frac{1}{\sqrt{2}}\sum_{n=0}^{\infty}u^i_{(n)}(x_2)\varphi_{(n)}(x^m,\theta_2)\,. \tag{3.16}$$

Here we have chosen the normalization of $u^i_{(n)}(x_2)$ as[4]

$$\int\! {\rm d}x_2\ {\rm Re}\left\{\bar{u}_{(n)i}(x_2)u^i_{(m)}(x_2)\right\}=\delta_{nm}\,. \tag{3.17}$$

By substituting eq. (3.16) into eq. (3.8) and performing the x_2-integration, the original $4D$ action (3.1) can be rewritten in terms of infinite $3D$ superfields as follows.

$$S=\int\! {\rm d}^3x{\rm d}^2\theta_2\ \left[\sum_{n=0}^{\infty}\left\{\frac{1}{2}(D_2\varphi_{(n)})^2+m_{(n)}\varphi^2_{(n)}\right\}+(\text{interaction terms})\right]. \tag{3.18}$$

Finally, by integrating out the heavy modes, we can derive the low-energy effective action on the domain wall.

4. Modified mode-expansion

4.1 Nambu-Goldstone mode

In the expression (3.18), the Q_2-SUSY is manifest since it is written in terms of superfields for the Q_2-SUSY. However, the nonlinear Q_1-SUSY is not unclear. In fact, the Q_1-transformation of each mode $\varphi_{(n)}$ defined by the mode-expansion (3.16) does not close on $\varphi_{(n)}$.

[4]We have assumed eigenvalues $m_{(n)}$ to be real. See [12, appendix C].

Considering eq. (3.16) and the Q_1-transformation of the $4D$ field φ^i, which is derived in appendix C, we can extract the Q_1-transformation law of $\varphi_{(n)}$ as follows.

$$\delta_{\xi_1}^{Q_1}\varphi_{(n)} = -2\kappa^{-1}\xi_1\theta_2\delta_{n,0} - 2\xi_1\theta_2\sum_m U_{nm}\varphi_{(m)} + \sum_m V_{nm}\xi_1 D_2\varphi_{(m)}\,, \tag{4.1}$$

where

$$\begin{aligned}\kappa^{-2} &\equiv \int \mathrm{d}x_2\,\left\{K_{i\bar{j}}(A_{\mathrm{cl}})\partial_2 A_{\mathrm{cl}}^i\partial_2\bar{A}_{\mathrm{cl}}^{\bar{j}} + K^{i\bar{j}}(A_{\mathrm{cl}})W_i(A_{\mathrm{cl}})\bar{W}_{\bar{j}}(\bar{A}_{\mathrm{cl}})\right\} \\ &= 2\int \mathrm{d}x_2\, K_{i\bar{j}}(A_{\mathrm{cl}})\partial_2 A_{\mathrm{cl}}^i\partial_2\bar{A}_{\mathrm{cl}}^{\bar{j}}\end{aligned} \tag{4.2}$$

is the tension of the domain wall, and

$$\begin{aligned}U_{nm} &\equiv \int \mathrm{d}x_2\,\mathrm{Re}\left\{\bar{u}_{(n)i}\partial_2 u_{(m)}^i\right\}\,, \\ V_{nm} &\equiv \int \mathrm{d}x_2\,\mathrm{Im}\left\{\bar{u}_{(n)i}u_{(m)}^i\right\}\,.\end{aligned} \tag{4.3}$$

Here we have used the fact that

$$\frac{1}{\sqrt{2}}u_{(0)}^i(x_2) = \kappa\partial_2 A_{\mathrm{cl}}^i(x_2)\,. \tag{4.4}$$

We can derive the P_2-transformation law of $\varphi_{(n)}$ in a similar way.

$$\delta_a^{P_2}\varphi_{(n)} = a\left(\kappa^{-1}\delta_{n,0} + \sum_m U_{nm}\varphi_{(m)}\right)\,. \tag{4.5}$$

As mentioned above, either transformation (4.1) or (4.5) does not close on each $\varphi_{(n)}$.

Next, we will modify the mode-expansion (3.16) so that the broken symmetries Q_1 and P_2 are nonlinearly realized on each mode. Consider the following mode-expansion.

$$\varphi^i(x^m, x_2, \theta_2) = A_{\mathrm{cl}}^i(x_2+\phi_0) + \frac{1}{\sqrt{2}}\sum_{n\neq 0} u_{(n)}^i(x_2+\phi_0)\varphi_{(n)}(x^m,\theta_2)\,, \tag{4.6}$$

where ϕ_0 is a *complex* scalar function of the NG mode φ_0. From this mode-expansion and eqs. (C.1) and (C.2) in appendix C, the transformation laws of ϕ_0 are read off as

$$\begin{aligned}\delta_{\xi_1}^{Q_1}\phi_0 &= -2\xi_1\theta_2 - i\xi_1 D_2\phi_0\,, \\ \delta_a^{P_2}\phi_0 &= a\,.\end{aligned} \tag{4.7}$$

These coincide with eqs. (2.22) and (2.23). Therefore, ϕ_0 defined by the modified mode-expansion (4.6) can be identified with ϕ_0 in eq. (2.21) in the previous section.

Indeed, by substituting eq. (4.6) into eq. (3.8), dropping the massive modes $\varphi_{(n)}$ ($n\neq 0$), and carrying out the x_2-integration, we will reproduce the supersymmetric Nambu-Goto action eq. (2.13).

4.2 Matter action

Although the NG mode φ_0 is identified with that of the nonlinear realization in the mode-expansion (4.6), the Q_1-transformation law of the other modes $\varphi_{(n)}$ $(n \neq 0)$ does not close on themselves. Then, in this subsection, we will further modify the mode-expansion so that $\varphi_{(n)}$ $(n \neq 0)$ transforms in the standard nonlinear transformation under the Q_1-SUSY.

Before we proceed, let us comment on the validity of such modification. From the $3D$ viewpoint, the modification of mode-expansion from eq. (3.16) to eq. (4.6) corresponds to the redefinition of the superfield $\varphi_{(n)}$. Note that such field redefinition involves space-time derivatives. In fact, although the original theory (3.1) contains no derivative couplings, the resulting effective theory has derivative couplings. (See eq. (2.13) or (B.4).) Hence, this field redefinition induces a new cut-off scale into the theory, which is $f \equiv \kappa^{-2/3}$.

Naively thinking, the mass of the first excited mode is thought to be $\mathcal{O}(f)$, and thus all modes except the NG mode φ_0 in eq. (4.6) should be integrated out. However, there can be exist modes lighter than the cut-off scale f in some models. For example, the theory with

$$\begin{aligned} K(\Phi,\bar{\Phi},X,\bar{X}) &= \Phi\bar{\Phi} + X\bar{X}\,, \\ W(\Phi,X) &= \Lambda^2\Phi - \frac{g}{3}\Phi^3 - h\Phi X^2\,, \qquad (\Lambda,g,h>0) \end{aligned} \tag{4.8}$$

has the following BPS domain wall.

$$\begin{aligned} A_{\rm cl}^{\Phi}(x_2) &= \frac{\Lambda}{\sqrt{g}}\tanh(\sqrt{g}\Lambda x_2)\,, \\ A_{\rm cl}^{X}(x_2) &= 0\,. \end{aligned} \tag{4.9}$$

In this case, the dynamical scale of the domain wall f is

$$f = \left(\frac{8}{3\sqrt{g}}\right)^{1/3}\Lambda\,, \tag{4.10}$$

and a "matter field" X contains modes with the mass eigenvalues [16, 17]

$$m_{(n)} = \sqrt{n\left(\frac{4h}{g}-n\right)}\sqrt{g}\Lambda\,, \qquad \left(n = 0,1,2,\dots < \frac{2h}{g}\right). \tag{4.11}$$

This means that in the case of $g \ll h < 1$, which corresponds to the fat brane [18], there are many light modes which satisfy the condition $m_{(n)} \ll f$. In the brane-world model-building, such light modes, especially the massless modes besides the NG mode, play important roles. Therefore, the further modification of the mode-expansion involving the "matter" modes $\varphi_{(n)}$ $(n \neq 0)$ is a useful work.

The modified mode-expansion is

$$\varphi^i = A_{\rm cl}^i(x_2+\phi_0) + \frac{1}{\sqrt{2}}\sum_{n\neq 0} u_{(n)}^i(x_2+\phi_0)\tilde{\varphi}_{(n)}(x^m,\theta_2)\,, \tag{4.12}$$

with

$$\tilde{\varphi}_{(n)} \equiv \varphi_{(n)} - i\kappa\zeta_0 D_2\varphi_{(n)} - \kappa^2\zeta_0^\alpha D_{2\alpha}\zeta_0^\beta D_{2\beta}\varphi_{(n)} + \frac{\kappa^2}{4}\zeta_0^2 D_2^2\varphi_{(n)} + \mathcal{O}(\kappa^3)\,. \tag{4.13}$$

Here ζ_0 is defined by eq. (2.16). In this case, each mode transforms under the broken symmetries as follows.

$$\begin{aligned} \delta_a^{P_2}\varphi_{(n)} &= 0\,, \\ \delta_{\xi_1}^{Q_1}\varphi_{(n)} &= -i\kappa\xi_1\gamma_{(3)}^m\zeta_0\partial_m\varphi_{(n)} + \mathcal{O}(\kappa^2)\,. \end{aligned} \tag{4.14}$$

These coincide with eqs. (2.3) and (2.4) up to $\mathcal{O}(\kappa)$.

In fact, by substituting eq. (4.12) into eq. (3.8), we can obtain the following lagrangian after somewhat tedious calculation.[5]

$$\begin{aligned} \mathcal{L}^{(3)} = \int\mathrm{d}^2\theta_2 2\Xi + \int\mathrm{d}^2\theta_2 E\Bigg[&\sum_{n\neq 0}\left\{\frac{1}{2}\left(\mathcal{D}_2\varphi_{(n)}\right)^2 + m_{(n)}\varphi_{(n)}^2\right\} + V(\varphi_{(n)}) + \\ &+\kappa\sum_{n,m} g_{(nm)}\varphi_{(m)}\left(\mathcal{D}_2^2\zeta_0\mathcal{D}_2\varphi_{(n)} + 2i\mathcal{D}_2\gamma_{(3)}^m\zeta_0\mathcal{D}_m\varphi_{(n)}\right) + \\ &+\kappa\mathcal{D}_2^\alpha\zeta_{0\alpha}U(\varphi_{(n)}) + \kappa^2\left(\frac{1}{2}\mathcal{D}_2^\alpha\zeta_0^\beta\mathcal{D}_{2\alpha}\zeta_{0\beta} + \mathcal{D}_2^\alpha\zeta_0^\beta\mathcal{D}_{2\beta}\zeta_{0\alpha}\right)\times \\ &\times\left(\mathcal{D}_2\varphi_{(n)}\right)^2 + \kappa^2\mathcal{D}_2^\alpha\zeta_0^\beta\mathcal{D}_{2\beta}\zeta_{0\alpha}\tilde{V}(\varphi_{(n)})\Bigg] + \mathcal{O}(\kappa^3)\,, \end{aligned} \tag{4.15}$$

where E is defined by eq. (2.15), and

$$\begin{aligned} g_{(nm)} &\equiv \int\mathrm{d}x_2\,\mathrm{Im}\left(\bar{u}_{(n)i}u_{(m)}^i\right)\,, \\ V &\equiv \sum_{N=3}^\infty\sum_{n_1,\ldots,n_N}\left\{4\int\mathrm{d}x_2\,\mathrm{Im}\left(\frac{e^{-i\delta}}{N!}u_{(n_1)}^{i_1}\cdots u_{(n_N)}^{i_N}W_{i_1\ldots i_N}\right)\right\}\varphi_{(n_1)}\cdots\varphi_{(n_N)}\,, \\ \tilde{V} &\equiv \sum_{N=1}^\infty\sum_{n_1,\ldots,n_N}\left\{4\int\mathrm{d}x_2\mathrm{Im}\left(\frac{e^{-i\delta}}{N!}u_{(n_1)}^{i_1}\cdots u_{(n_N)}^{i_N}W_{i_1\ldots i_N}\right)\right\}\varphi_{(n_1)}\cdots\varphi_{(n_N)}\,, \\ U &\equiv \sum_{N=2}^\infty\sum_{n_1,\ldots,n_N}\left\{4\int\mathrm{d}x_2\mathrm{Re}\left(\frac{e^{-i\delta}}{N!}u_{(n_1)}^{i_1}\cdots u_{(n_N)}^{i_N}W_{i_1\ldots i_N}\right)\right\}\varphi_{(n_1)}\cdots\varphi_{(n_N)}\,. \end{aligned} \tag{4.16}$$

The above lagrangian certainly has an invariance under the Q_1-SUSY, which is non-linearly realized. The first term is the supersymmetric Nambu-Goto lagrangian, and the remaining part corresponds to the matter lagrangian.

5. Broken Lorentz and $\mathrm{U}(1)_R$ symmetries

In the matter lagrangian in eq. (4.15), note that the NG mode φ_0 appears not only through E and $\mathcal{D}_2$, but also in the form such as $\mathcal{D}_2^\alpha\zeta_{0\beta}$. This means that even if we know the lagrangian in the limit of $\kappa \to 0$ in some way, we cannot reproduce the full

[5]For simplicity, we have assumed the minimal Kähler potential.

lagrangian eq. (4.15) by using the method of the nonlinear realization. This stems from the fact that we did not respect the broken Lorentz symmetry and the $\mathrm{U}(1)_R$ symmetry in the modification of the mode-expansion (4.12) and (4.13).

In the nonlinear realization for space-time symmetries, if we take into account only the (super-)translational generators as generators of the whole bulk symmetry G, there will be an ambiguity of inserting a dimensionless tensor, such as $\kappa\mathcal{D}_2^\alpha\zeta_{0\beta}$, into the G-invariant effective action [11]. For example, let us assume that the lagrangian at $\kappa\to 0$ is[6]

$$\mathcal{L}^{(3)}_{\kappa\to 0} = \int d^2\theta_2 \sum_{n\neq 0}\left\{\frac{1}{2}\left(D_2\varphi_{(n)}\right)^2 + m_{(n)}\varphi_{(n)}^2\right\} + V\left(\varphi_{(n)}\right). \tag{5.1}$$

Then, the most general lagrangian that is P_2- and Q_1-invariant is written as

$$\begin{aligned}\mathcal{L}^{(3)} = &\int d^2\theta_2\, E\Bigg[\sum_{n\neq 0}\left\{\frac{1}{2}\left(\mathcal{D}_2\varphi_{(n)}\right)^2 + m_{(n)}\varphi_{(n)}^2\right\} + V\left(\varphi_{(n)}\right) + \\ &\qquad + \mathcal{F}\left(\mathcal{D}_2^\alpha\zeta_{0\beta}, \mathcal{D}_m\zeta_{0\alpha}, \mathcal{D}_2^\alpha\mathcal{D}_2^\beta\zeta_{0\gamma},\ldots\right)\Bigg],\end{aligned} \tag{5.2}$$

where $\mathcal{F}$ denotes $3D$ Lorentz-invariant terms that include at least one covariant derivative of ζ_0. Note that the above lagrangian has an ambiguity of adding terms including arbitrary numbers of $\kappa\mathcal{D}_2^\alpha\zeta_{0\beta}$ since such tensor is dimensionless. In fact, the second and the third lines in eq. (4.15) correspond to $\mathcal{F}$ in eq. (5.2), and cannot be determined by the nonlinear realization.

The above ambiguity already exists in the definition of $\tilde{\varphi}_{(n)}$ in eq. (4.13). We may add terms, such as $\kappa\mathcal{D}_2^\alpha\zeta_{0\alpha}\varphi_{(n)}$, to the definition of $\tilde{\varphi}_{(n)}$ since the resulting transformation laws of $\varphi_{(n)}$ do not change. Such terms contribute to $\mathcal{F}$ in eq. (5.2).

In order to remove such ambiguity, we should take the full symmetry group G as

$$G_{\max} = \{P_m, P_2, Q_{1\alpha}, Q_{2\alpha}, M_{mn}, K_m, R\}, \tag{5.3}$$

which contains the maximal automorphism group of the $4D$ $\mathcal{N}=1$ SUSY algebra. Here M_{mn} and $K_m \equiv M_{m2}$ denote the $4D$ Lorentz generators, and R is a generator of $\mathrm{U}(1)_R$. In this case, the vacuum stability subgroup is

$$H_{\max} = \{P_m, Q_{2\alpha}, M_{mn}\}. \tag{5.4}$$

The algebra among these generators is listed in eq. (D.1) in appendix D.1.

The transformation laws of ϕ_0 in the nonlinear realization under the K- and R-transformations can be defined by

$$\delta_v^K\phi_0 = v^m\left(-x_m - \phi_0\partial_m\phi_0 + \frac{1}{2}\theta_2\gamma_{(3)m}D_2\phi_0\right), \tag{5.5}$$

$$\delta_r^R\phi_0 = r\left(\theta_2^2 + i\theta_2 D_2\phi_0\right), \tag{5.6}$$

[6] In fact, such lagrangian can be calculated much easier than eq. (4.15) since eqs. (4.12) and (4.13) become very simple forms by dropping φ_0 (and thus ζ_0) from eq. (4.12).

where v^m and r are transformation parameters. These form the SUSY algebra (D.1) together with eqs. (2.22) and (2.23).

From eq. (5.5),

$$\delta_v^K \Xi = v^m \left\{ -\kappa \partial_m (\varphi_0 \Xi) + \frac{i}{2} \kappa^{-1} D_2^\alpha \left((\gamma_{(3)m} \theta_2)_\alpha \varphi_0 \right) \right\} . \tag{5.7}$$

This is a total derivative and thus the NG action $S_{\rm NG}$ in eq. (2.13) is invariant under the K-transformation.

For the $\mathrm{U}(1)_R$ symmetry, on the other hand, we can see from eq. (5.6),

$$\delta_r^R \Xi = \kappa^{-1} r \theta_2 D_2 \varphi_0 . \tag{5.8}$$

This means

$$\delta_r^R S_{\rm NG} = \kappa^{-1} r \int \mathrm{d}^3 x 2 f_0 , \tag{5.9}$$

where f_0 is the auxiliary field of φ_0. So $S_{\rm NG}$ does not have the off-shell $\mathrm{U}(1)_R$ symmetry. However, considering the fact that $f_0 = 0$ on shell as mentioned in appendix B, we can see that $S_{\rm NG}$ is R-invariant *on shell*.

As a result, $S_{\rm NG}$ obtained in the nonlinear realization is invariant under the full $G_{\rm max}$ symmetry.

On the other hand, the NG action obtained by the mode-expansion (4.12) does not have an invariance under the broken Lorentz symmetry. Although it has the same form as eq. (2.13), the K-transformation of ϕ_0 does not coincide with eq. (5.5). Unlike the nonlinear realization, the transformation laws of each mode are determined from those of the bulk theory in the mode-expansion approach. The K-transformation law of φ^i is listed in eq. (C.3) in appendix C. Due to the explicit appearance of x_2 in eq. (C.3), the K-transformation of ϕ_0 does not close on ϕ_0.

Now we will try to modify the mode-expansion (4.12) further, so that ϕ_0 transforms properly under all broken symmetries $G_{\rm max}/H_{\rm max}$.

We propose the following mode-expansion.

$$\varphi^i(x^m, x_2, \theta_2) = A^i_{\rm cl}(\hat{x}_2) + \frac{1}{\sqrt{2}} \sum_{n \neq 0} u^i_{(n)}(\hat{x}_2) \tilde{\varphi}_{(n)}(\hat{x}^m, \theta_2) , \tag{5.10}$$

where

$$\begin{aligned} \hat{x}^m &\equiv x^m - x_2 \partial^m \phi_0 - \frac{\kappa^2}{2} \partial^m (\varphi_0^2) + \frac{\kappa^2}{2} x_2 \partial^m (\partial^n \varphi_0 \partial_n \varphi_0) + \mathcal{O}(\kappa^3) , \\ \hat{x}_2 &\equiv x_2 + \phi_0 - \frac{\kappa^2}{2} x_2 \partial^n \varphi_0 \partial_n \varphi_0 + \mathcal{O}(\kappa^3) , \end{aligned} \tag{5.11}$$

and $\tilde{\varphi}_{(n)}$ is the one defined in eq. (4.13).

With this mode-expansion and eq. (C.3), we can derive the K-transformation of each mode.

$$\delta_v^K \phi_0 = v^m \left(-x_m + \frac{1}{2} \theta_2 \gamma_{(3)m} D_2 \phi_0 \right) + \mathcal{O}(\kappa^2) , \tag{5.12}$$

$$\delta_v^K \tilde{\varphi}_{(n)} = v^m \left(-\phi_0 \partial_m \tilde{\varphi}_{(n)} + \frac{1}{2} \theta_2 \gamma_{(3)m} D_2 \tilde{\varphi}_{(n)} \right) + \mathcal{O}(\kappa^2) . \tag{5.13}$$

eq. (5.12) coincides with eq. (5.5) up to $\mathcal{O}(\kappa)$. Considering the definition of $\tilde{\varphi}_{(n)}$ in eq. (4.13), eq. (5.13) is translated into

$$\delta_v^K \varphi_{(n)} = v^m \left\{ -\kappa\varphi_0 \partial_m \varphi_{(n)} + \frac{\kappa}{2}\zeta_0 \gamma_{(3)m}\theta_2 \partial_m \varphi_{(n)} + \frac{i}{2}\kappa\zeta_0\gamma_{(3)m} D_2 \varphi_{(n)} \right\} + \mathcal{O}(\kappa^2)\,. \quad (5.14)$$

Noticing that $\varphi_0 = \rho_0 + \mathcal{O}(\kappa^2)$, this coincides with the standard nonlinear transformation eq. (D.5) in appendix D.2.

In a similar way, we can derive the transformation laws for other symmetries.

For the broken SUSY, we obtain

$$\begin{aligned} \delta_{\xi_1}^{Q_1}\phi_0 &= -2\xi_1\theta_2 - i\xi_1 D_2\phi_0 + \mathcal{O}(\kappa^2)\,, \\ \delta_{\xi_1}^{Q_1}\varphi_{(n)} &= -i\kappa\xi_1\gamma_{(3)}^m \zeta_0 \partial_m \varphi_{(n)} + \mathcal{O}(\kappa^2)\,. \end{aligned} \quad (5.15)$$

For the $\mathrm{U}(1)_R$ symmetry, by using eq. (C.4) in appendix C, we obtain

$$\begin{aligned} \delta_r^R \phi_0 &= r(\theta_2^2 + i\theta_2 D_2\phi_0) + \mathcal{O}(\kappa^2)\,, \\ \delta_r^R \varphi_{(n)} &= -i\kappa r\zeta_0\gamma_{(3)}^m \theta_2 \partial_m \varphi_{(n)} - \kappa r\zeta_0 D_2 \varphi_{(n)} + \mathcal{O}(\kappa^2)\,. \end{aligned} \quad (5.16)$$

Thus, ϕ_0 transforms in the desired way in the case of our modified mode-expansion (5.10) and (5.11). Note that eqs. (5.15) and (5.16) are the standard nonlinear transformations, which are listed in eq. (D.4) and (D.6) in appendix D.2. Therefore, the modified mode-expansion (5.10) and (5.11) defines modes on which all the broken symmetries are nonlinearly realized at least up to $\mathcal{O}(\kappa)$.

6. Summary and discussion

There are mainly two different approaches to derive the low-energy effective theory in the background of a BPS domain wall. Each approach has its own advantages and disadvantages.

The first one is the nonlinear realization approach. We can construct an effective action on the wall that is invariant under both the broken and the unbroken SUSYs by using the nonlinear realization technique. This approach is useful when we discuss the general properties of BPS brane-like objects, such as BPS walls or supermembranes. However, in this approach, we neglect the wall width and cannot discuss the specific wall profile. Namely, we cannot determine parameters of the effective theory by this approach.

The second one is the mode-expansion approach. In this approach, we start directly from the bulk theory. So we need to specify the bulk theory and thus the results are model-dependent. Since we explicitly derive the effective theory from the bulk theory, the relation between the bulk $4D$ superfields and the $3D$ superfields in the effective theory is clear in this approach. On the other hand, SUSY broken by the wall is not respected since its transformation of each mode does not close on itself.

Therefore, it is very useful and instructive to clarify the relation between the above two approaches.

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In this paper, we proposed a modified mode-expansion so that the broken SUSY is nonlinearly realized on each mode. Indeed, our mode-expansion leads to a supersymmetric Nambu-Goto action for the NG mode, and to a matter action with a form expected from the nonlinear realization for the other modes. In particular, the NG mode φ_0 defined by our mode-expansion can be identified with that of the nonlinear realization *for all orders* in κ. For the other modes, their transformation law under the broken SUSY coincides with the standard nonlinear transformation up to $\mathcal{O}(\kappa)$.

Note that our modification of the mode-expansion corresponds to the redefinition of the $3D$ superfields. This field redefinition involves space-time derivatives and the scale $f \equiv \kappa^{-2/3}$. So the cut-off scale of the effective theory becomes f after the field redefinition.

We also showed that it is possible to modify the mode-expansion so that each mode transforms in the standard nonlinear transformation under not only the broken SUSY but also the broken Lorentz and $\mathrm{U}(1)_R$ symmetries at least up to $\mathcal{O}(\kappa)$. However, whether the extension to higher orders in κ is possible is not clear to us.

In this paper, we have discussed the BPS domain wall. When we construct a realistic brane-world model in a SUSY theory, we must consider the SUSY breaking mechanism. One of the simplest mechanism of SUSY breaking is the coexistence of the BPS and anti-BPS walls [16]. In such a case, each domain wall preserves an opposite half of the bulk SUSY, and all of the supersymmetries are broken in the whole system. In the thin wall limit, this corresponds to the one called the *pseudo-supersymmetry* [19]. The author of ref. [19] derives the effective theory of the brane-antibrane system by using the nonlinear realization technique. In this case, the SUSY breaking effects are induced at loop level because tree-level couplings between the branes are absent. For the wall-antiwall system with a finite wall-width, on the other hand, SUSY breaking appears at tree level although its effects are exponentially suppressed for the distance between the walls [16]. To discuss the phenomenological arguments, it is useful to describe the effective theory on the wall in terms of the superfields and the SUSY breaking terms. Combining the method in Ref.[19] and the result of this paper, we can derive the effective theory in the wall-antiwall system in terms of the $3D$ superfields and the SUSY breaking terms. This work is now in progress.

Acknowledgments

The author thanks the Yukawa Institute for Theoretical Physics at Kyoto University, where this work was initiated during the YITP-W-99-99 on "Extra dimensions and Braneworld". The author also thanks Koji Hashimoto for useful discussion.

A. Notations

Basically, we follow the notations of ref. [20] for the $4D$ bulk theory. The notations for the $3D$ theories are as follows.

We take the space-time metric as

$$\eta^{mn} = \mathrm{diag}(-1, +1, +1)\,. \tag{A.1}$$

The $3D$ γ-matrices, $(\gamma^m_{(3)})_\alpha^{\ \beta}$, can be written by the Pauli matrices as

$$\gamma^0_{(3)} = \sigma^2\,, \qquad \gamma^1_{(3)} = -i\sigma^3\,, \qquad \gamma^3_{(3)} = i\sigma^1\,, \tag{A.2}$$

and these satisfy the $3D$ Clifford algebra,

$$\left\{\gamma^m_{(3)}, \gamma^n_{(3)}\right\} = -2\eta^{mn}\,. \tag{A.3}$$

The spinor indices are raised and lowered by multiplying σ^2 from the left.

$$\psi_\alpha = (\sigma^2)_{\alpha\beta}\psi^\beta\,, \qquad \psi^\alpha = (\sigma^2)^{\alpha\beta}\psi_\beta\,. \tag{A.4}$$

We take the following convention of the contraction of spinor indices.

$$\psi_1\psi_2 \equiv \psi_1^\alpha\psi_{2\alpha} = (\sigma^2)_{\alpha\beta}\psi_1^\alpha\psi_2^\beta = \psi_2\psi_1\,. \tag{A.5}$$

A.1 Covariant derivatives

The algebra of the $3D$ $\mathcal{N}=1$ SUSY preserved by the wall is

$$\{Q_{2\alpha}, Q_{2\beta}\} = 2(\gamma^m_{(3)}\sigma^2)_{\alpha\beta}P_m\,. \tag{A.6}$$

The representation of the generators on the $3D$ $\mathcal{N}=1$ superspace (x^m, θ_2) is

$$\begin{aligned}\hat{P}_m &= -i\partial_m\,,\\ \hat{Q}_{2\alpha} &= \partial_{2\alpha} + i(\gamma^m_{(3)}\theta_2)_\alpha\partial_m\,.\end{aligned} \tag{A.7}$$

The SUSY covariant derivative for θ_2 is

$$D_{2\alpha} \equiv \partial_{2\alpha} - i(\gamma^m_{(3)}\theta_2)_\alpha\partial_m\,. \tag{A.8}$$

We list the covariant derivatives for superspace coordinates in the presence of the NG superfields as follows. They can be obtained by calculating the Cartan one-form $\Omega^{-1}\mathrm{d}\Omega$ where Ω is defined in eq. (2.2).

$$\begin{aligned}\mathcal{D}_m\rho_0 &= (\omega^{-1})_m^{\ n}\partial_n\rho_0 = \partial_m\rho_0 + \mathcal{O}(\kappa^2)\,,\\ \mathcal{D}_{2\alpha}\rho_0 &= D_{2\alpha}\rho_0 + 2\zeta_{0\alpha} - i\kappa^2 D_{2\alpha}\zeta_0\gamma^m_{(3)}\zeta_0(\omega^{-1})_m^{\ n}\partial_n\rho_0\,,\\ \mathcal{D}_m\zeta_{0\beta} &= (\omega^{-1})_m^{\ n}\partial_n\zeta_{0\beta} = \partial_m\zeta_{0\beta} + \mathcal{O}(\kappa^2),\\ \mathcal{D}_{2\alpha}\zeta_{0\beta} &= D_{2\alpha}\zeta_{0\beta} - i\kappa^2 D_{2\alpha}\zeta_0\gamma^m_{(3)}\zeta_0(\omega^{-1})_m^{\ n}\partial_n\zeta_{0\beta}\,,\\ \mathcal{D}_m\phi &= (\omega^{-1})_m^{\ n}\partial_n\phi = \partial_m\phi + \mathcal{O}(\kappa^2)\,,\\ \mathcal{D}_{2\alpha}\phi &= D_{2\alpha}\phi - i\kappa^2 D_{2\alpha}\zeta_0\gamma^m_{(3)}\zeta_0(\omega^{-1})_m^{\ n}\partial_n\phi\,,\end{aligned} \tag{A.9}$$

where ϕ denotes a matter field, and

$$\omega_m^{\ n} \equiv \delta_m^{\ n} + i\kappa^2\partial_m\zeta_0\gamma^n_{(3)}\zeta_0\,. \tag{A.10}$$

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A.2 Action of the generators on the fields

The SUSY transformation δ^Q_ξ of a chiral supermultiplet (A, Ψ^α, F) is defined by

$$\begin{aligned}
\delta^Q_\xi A &= \sqrt{2}\xi\Psi\,, \\
\delta^Q_\xi \Psi_\alpha &= i\sqrt{2}(\sigma^\mu\bar{\xi})_\alpha \partial_\mu A + \sqrt{2}\xi_\alpha F\,, \\
\delta^Q_\xi F &= i\sqrt{2}\bar{\xi}\bar{\sigma}^\mu\partial_\mu\Psi\,.
\end{aligned} \tag{A.11}$$

We define an action of the generators P_μ, Q_α and $\bar{Q}^{\dot{\alpha}}$ on the fields $\phi = A, \Psi^\alpha, F$ as

$$\begin{aligned}
P_\mu \times \phi &\equiv -i\partial_\mu\phi\,, \\
(\xi Q + \bar{\xi}\bar{Q}) \times \phi &\equiv \delta^Q_\xi \phi\,.
\end{aligned} \tag{A.12}$$

Under the above definition, the chiral superfield Φ can be written as

$$\Phi(x, \theta, \bar{\theta}) = e^{ix^\mu P_\mu + \theta Q + \bar{\theta}\bar{Q}} \times A(0)\,. \tag{A.13}$$

There is a useful formula that converts covariant derivatives $\mathcal{D}_M$ into the corresponding generators Γ_M.

$$\mathcal{D}_M e^{iX^N\Gamma_N} = e^{iX^N\Gamma_N}\Gamma_M\,, \tag{A.14}$$

where X^M are coordinates for Γ_M.

For the SUSY generator Q_2, for instance,

$$\mathcal{D}_{2\alpha} e^{ix^m P_m + \theta_2 Q_2} = e^{ix^m P_m + \theta_2 Q_2} Q_{2\alpha}\,. \tag{A.15}$$

Using this formula, we can rewrite the chiral condition for Φ as follows.

$$\begin{aligned}
\bar{D}_{\dot{\alpha}}\Phi &= \bar{D}_{\dot{\alpha}} e^{\theta Q + \bar{\theta}\bar{Q}} \times A(x) \\
&= e^{\theta Q + \bar{\theta}\bar{Q}} \bar{Q}_{\dot{\alpha}} \times A(x) = 0\,.
\end{aligned} \tag{A.16}$$

Then,

$$\bar{Q}_{\dot{\alpha}} \times A(x) = -\frac{e^{i\delta/2}}{\sqrt{2}}(Q_{1\alpha} + iQ_{2\alpha}) \times A(x) = 0\,. \tag{A.17}$$

Namely,

$$Q_{1\alpha} \times A = -iQ_{2\alpha} \times A\,. \tag{A.18}$$

B. Bosonic part of $S_{\rm NG}$

Here we will provide a brief derivation of the Nambu-Goto action from the effective action $S_{\rm NG}$ defined in eq. (2.13). We denote the component fields of φ_0 as

$$\varphi_0 = a_0 + \theta_2\eta_0 + \frac{1}{2}\theta_2 f_0\,. \tag{B.1}$$

In order to concentrate on the bosonic part of $S_{\rm NG}$, we will neglect the fermionic component η_0 in the following. Then,

$$\psi_0^2|_{\rm bosonic} = \frac{1}{4}\theta_2^2(f_0^2 - \partial^m a_0 \partial_m a_0)\,, \tag{B.2}$$

and

$$D_2^2(\psi_0^2)|_{\rm bosonic} = -f_0^2 + \partial^m a_0 \partial_m a_0 \,. \tag{B.3}$$

Therefore, the auxiliary field f_0 enters in $S_{\rm NG}$ only in a bilinear way. This means that an equation of motion for f_0 is f_0=0. Hence, after the elimination of f_0, the bosonic part of $S_{\rm NG}$ becomes

$$\begin{aligned} S_{\rm NG\ bosonic}^{\rm on\text{-}shell} &= \int {\rm d}^3x \frac{-\partial^m a_0 \partial_m a_0}{1+\sqrt{1+\kappa^2 \partial^m a_0 \partial_m a_0}} \\ &= \int {\rm d}^3x\, \kappa^{-2} \left\{ 1 - \sqrt{1+\kappa^2 \partial^m a_0 \partial_m a_0} \right\} . \end{aligned} \tag{B.4}$$

This is the Nambu-Goto action in the static gauge, as expected.

C. Transformation of φ^i under the broken symmetries

The transformation laws of φ^i for the broken symmetries are obtained from the definition of φ^i in eq. (3.9) and the SUSY algebra (2.1).

For example, for the Q_1-transformation,

$$\begin{aligned} \delta_{\xi_1}^{Q_1} \varphi^i &= \xi_1 Q_1 \times \varphi^i = \xi_1 Q_1 e^{ix^m P_m + ix_2 P_2 + \theta_2 Q_2} \times A^i(0) \\ &= e^{ix^m P_m + ix_2 P_2 + \theta_2 Q_2} (-2i\xi_1 \theta_2 P_2 + \xi_1 Q_1) \times A^i(0) \\ &= e^{ix^m P_m + ix_2 P_2 + \theta_2 Q_2} (-2i\xi_1 \theta_2 P_2 - i\xi_1 Q_2) \times A^i(0) \\ &= (-2\xi_1 \theta_2 \partial_2 - i\xi_1 D_2) \varphi^i \,. \end{aligned} \tag{C.1}$$

Here we have used the chiral condition eq. (A.18) and the formula eq. (A.14).

Similarly, for the P_2-transformation,

$$\delta_a^{P_2} \varphi^i = iaP_2 \times \varphi^i = a\partial_2 \varphi^i \,. \tag{C.2}$$

For the broken Lorentz transformation,

$$\delta_v^K \varphi^i = iv^m K_m \times \varphi^i = v^m \left\{ x_2 \partial_m \varphi^i - x_m \partial_2 \varphi^i + \frac{1}{2} \theta_2 \gamma_{(3)m} D_2 \varphi^i \right\} . \tag{C.3}$$

For the $\mathrm{U}(1)_R$ transformation,

$$\delta_r^R \varphi^i = irR \times \varphi^i = r \left\{ \theta_2^2 \partial_2 \varphi^i + i\theta_2 D_2 \varphi^i \right\} . \tag{C.4}$$

D. In the case of $G = G_{\rm max}$

Here we collect the main results of the nonlinear realization in the case that the full symmetry group G is taken to be $G_{\rm max} = \{P_m, P_2, Q_{1\alpha}, Q_{2\alpha}, M_{mn}, K_m, R\}$. The vacuum stability subgroup is $H_{\rm max} = \{P_m, Q_{2\alpha}, M_{mn}\}$ in this case.

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D.1 Super-Poincaré algebra

The $3D$ $\mathcal{N}=2$ super-Poincaré algebra with central extension is as follows.

$$
\begin{aligned}
\{Q_{1\alpha}, Q_{1\beta}\} &= \{Q_{2\alpha}, Q_{2\beta}\} = 2(\gamma^m_{(3)}\sigma^2)_{\alpha\beta} P_m \,, \\
\{Q_{1\alpha}, Q_{2\beta}\} &= -\{Q_{2\alpha}, Q_{1\beta}\} = 2i(\sigma^2)_{\alpha\beta} P_2 \,, \\
[M_{mn}, Q_{1\alpha}] &= -i\left(\gamma_{(3)mn} Q_1\right)_\alpha \,, \qquad [M_{mn}, Q_{2\alpha}] = -i\left(\gamma_{(3)mn} Q_2\right)_\alpha \,, \\
[K_m, Q_{1\alpha}] &= -\frac{1}{2}\left(\gamma_{(3)m} Q_2\right)_\alpha \,, \qquad [K_m, Q_{2\alpha}] = \frac{1}{2}\left(\gamma_{(3)m} Q_1\right)_\alpha \,, \\
[R, Q_{1\alpha}] &= -iQ_{2\alpha} \,, \qquad [R, Q_{2\alpha}] = iQ_{1\alpha} \,, \\
[M_{mn}, P_l] &= i(\eta_{lm} P_n - \eta_{ln} P_m) \,, \qquad [M_{mn}, P_2] = 0 \,, \\
[K_m, P_n] &= i\eta_{mn} P_2 \,, \qquad [K_m, P_2] = -iP_m \,, \\
[M_{mn}, M_{lp}] &= i(\eta_{ml} M_{np} - \eta_{nl} M_{mp} - \eta_{pn} M_{lm} + \eta_{pm} M_{ln}) \,, \\
[M_{mn}, K_l] &= i(\eta_{ml} K_n - \eta_{nl} K_m) \,, \\
[K_m, K_n] &= iM_{mn} \,.
\end{aligned} \tag{D.1}
$$

D.2 Standard nonlinear transformations

A coset element $\hat{\Omega}$ can be parameterized as

$$
\hat{\Omega} = e^{ix^m P_m + \theta_2 Q_2} e^{i\rho_0 P_2 + \zeta_0 Q_1} e^{i\lambda_0^m K_m + i\sigma_0 R} \,, \tag{D.2}
$$

where ρ_0, $\zeta_{0\alpha}$, λ_0^m and σ_0 are the NG superfields for the corresponding generators.

The transformation laws of each superfield are obtained by multiplying $\hat{\Omega}$ by corresponding group elements from the left.

For P_2-transformation,

$$
\begin{aligned}
\delta_a^{P_2} \rho_0 &= \kappa^{-1} a \,, \\
\delta_a^{P_2} \zeta_{0\alpha} &= \delta_a^{P_2} \lambda_0^m = \delta_a^{P_2} \sigma_0 = \delta_a^{P_2} \phi = 0 \,.
\end{aligned} \tag{D.3}
$$

For the broken SUSY,

$$
\begin{aligned}
\delta_{\xi_1}^{Q_1} \rho_0 &= -2\kappa^{-1} \xi_1 \theta_2 - i\kappa \xi_1 \gamma^m_{(3)} \zeta_0 \partial_m \rho_0 \,, \\
\delta_{\xi_1}^{Q_1} \zeta_{0\alpha} &= \kappa^{-1} \xi_{1\alpha} - i\kappa \xi_1 \gamma^m_{(3)} \zeta_0 \partial_m \zeta_{0\alpha} \,, \\
\delta_{\xi_1}^{Q_1} \lambda_0^m &= = -i\kappa \xi_1 \gamma^n_{(3)} \zeta_0 \partial_n \lambda_0^m \,, \\
\delta_{\xi_1}^{Q_1} \sigma_0 &= = -i\kappa \xi_1 \gamma^m_{(3)} \zeta_0 \partial_m \sigma_0 \,, \\
\delta_{\xi_1}^{Q_1} \phi &= = -i\kappa \xi_1 \gamma^m_{(3)} \zeta_0 \partial_m \phi \,.
\end{aligned} \tag{D.4}
$$

For the broken Lorentz symmetry,

$$
\begin{aligned}
\delta_v^K \rho_0 &= -\kappa^{-1} v^m x_m + \mathcal{O}(\kappa) \,, \\
\delta_v^K \zeta_{0\alpha} &= -\frac{i}{2}\kappa^{-1} v^m \left(\gamma_{(3)m}\theta_2\right)_\alpha + \mathcal{O}(\kappa) \,, \\
\delta_v^K \lambda_0^m &= \kappa^{-1} v^m + \mathcal{O}(\kappa) \,, \\
\delta_v^K \sigma_0 &= \mathcal{O}(\kappa) \,, \\
\delta_v^K \phi &= \kappa v^m \left\{ \left(-\rho_0 + \frac{1}{2}\zeta_0 \gamma^n_{(3)} \gamma_{(3)m} \theta_2\right) \partial_n \phi + \frac{i}{2}\zeta_0 \gamma_{(3)m} D_2 \phi \right\} \,.
\end{aligned} \tag{D.5}
$$

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For the $\mathrm{U}(1)_R$ symmetry,

$$
\begin{aligned}
\delta_r^R \rho_0 &= r \left\{ \kappa^{-1} \theta_2^2 - \kappa \zeta_0^2 - i\kappa \zeta_0 \gamma_{(3)}^m \theta_2 \partial_m \rho_0 - \kappa \zeta_0 D_2 \rho_0 \right\} , \\
\delta_r^R \zeta_{0\alpha} &= r \left\{ -\kappa^{-1} \theta_{2\alpha} - i\kappa \zeta_0 \gamma_{(3)}^m \theta_2 \partial_m \zeta_{0\alpha} - \kappa \zeta_0 D_2 \zeta_{0\alpha} \right\} , \\
\delta_r^R \lambda_0^m &= r \left\{ -i\kappa \zeta_0 \gamma_{(3)}^n \theta_2 \partial_n \lambda_0^m - \kappa \zeta_0 D_2 \lambda_0^m \right\} , \\
\delta_r^R \sigma_0 &= r \left\{ \kappa^{-1} - i\kappa \zeta_0 \gamma_{(3)}^m \theta_2 \partial_m \sigma_0 - \kappa \zeta_0 D_2 \sigma_0 \right\} , \\
\delta_r^R \phi &= r \left\{ -i\kappa \zeta_0 \gamma_{(3)}^m \theta_2 \partial_m \phi - \kappa \zeta_0 D_2 \phi \right\} .
\end{aligned}
\tag{D.6}
$$

Here, a, $\xi_{1\alpha}$, v^m and r are transformation parameters, and ϕ denotes a matter field.

D.3 Inverse Higgs effect

The NG superfields introduced in eq. (D.2) are not independent. The relation between them are obtained by the following covariant constraints [13, 21].

$$
\hat{\mathcal{D}}_{2\alpha} \rho_0 = 0 , \qquad \hat{\mathcal{D}}_{2\alpha} \zeta_0^\beta = 0 , \qquad \hat{\mathcal{D}}_m \rho_0 = 0 . \tag{D.7}
$$

Here $\hat{\mathcal{D}}_m$ and $\hat{\mathcal{D}}_{2\alpha}$ are covariant derivatives in the presence of the NG superfields, which are derived from the Cartan one-form $\hat{\Omega}^{-1} \mathrm{d}\hat{\Omega}$.

Solving these constraints, we can express all NG superfields in terms of a single superfield ρ_0.

$$
\begin{aligned}
\zeta_{0\alpha} &= -\frac{1}{2} D_{2\alpha} \rho_0 + \mathcal{O}(\kappa^2) , \\
\lambda_0^m &= i D_2 \gamma_{(3)}^m \zeta_0 + \mathcal{O}(\kappa^2) = -\partial^m \rho_0 + \mathcal{O}(\kappa^2) , \\
\sigma_0 &= \frac{1}{2} D_2^\alpha \zeta_{0\alpha} + \mathcal{O}(\kappa^2) = -\frac{1}{4} D_2^2 \rho_0 + \mathcal{O}(\kappa^2) .
\end{aligned}
\tag{D.8}
$$

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